An Introduction to Cryptography

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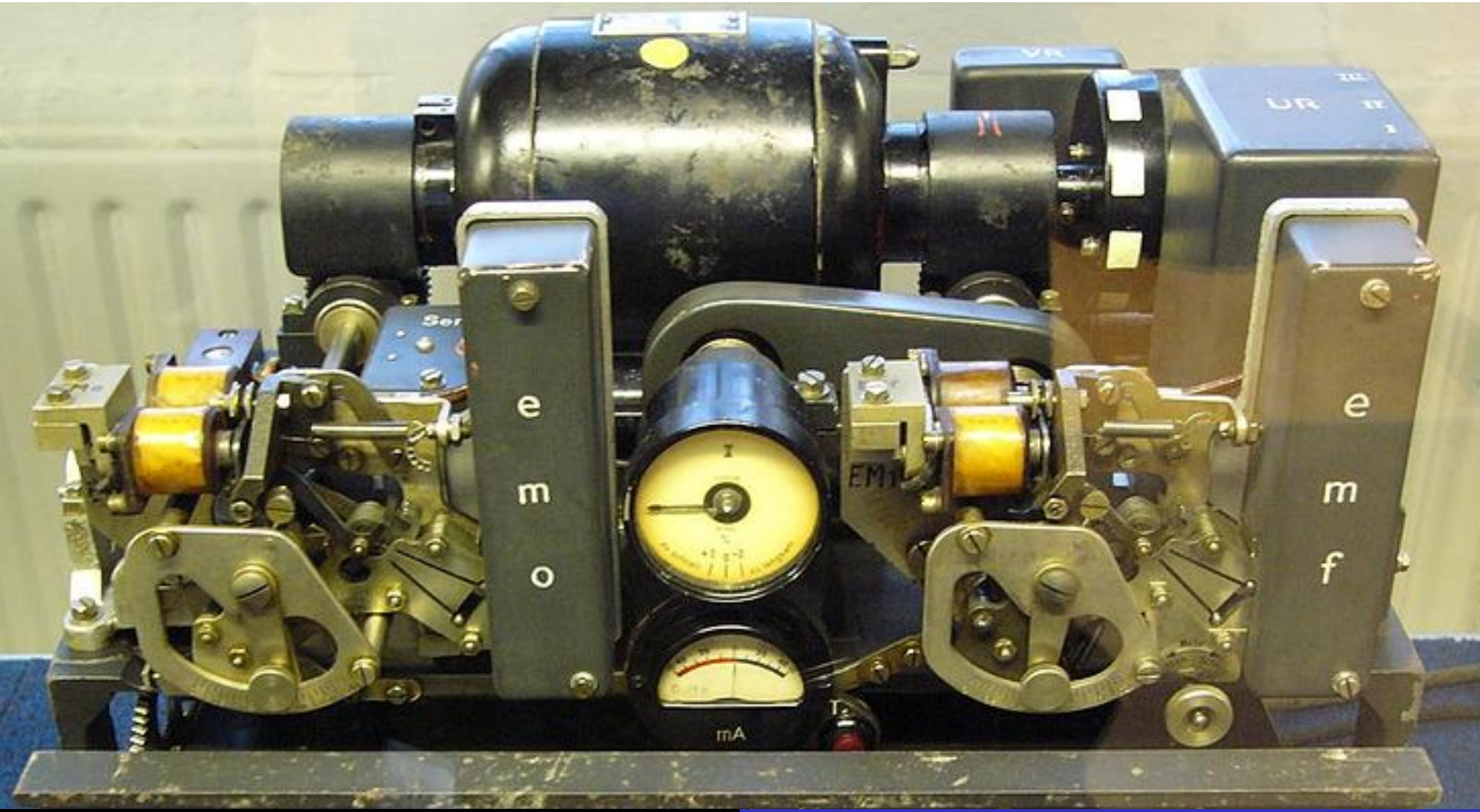
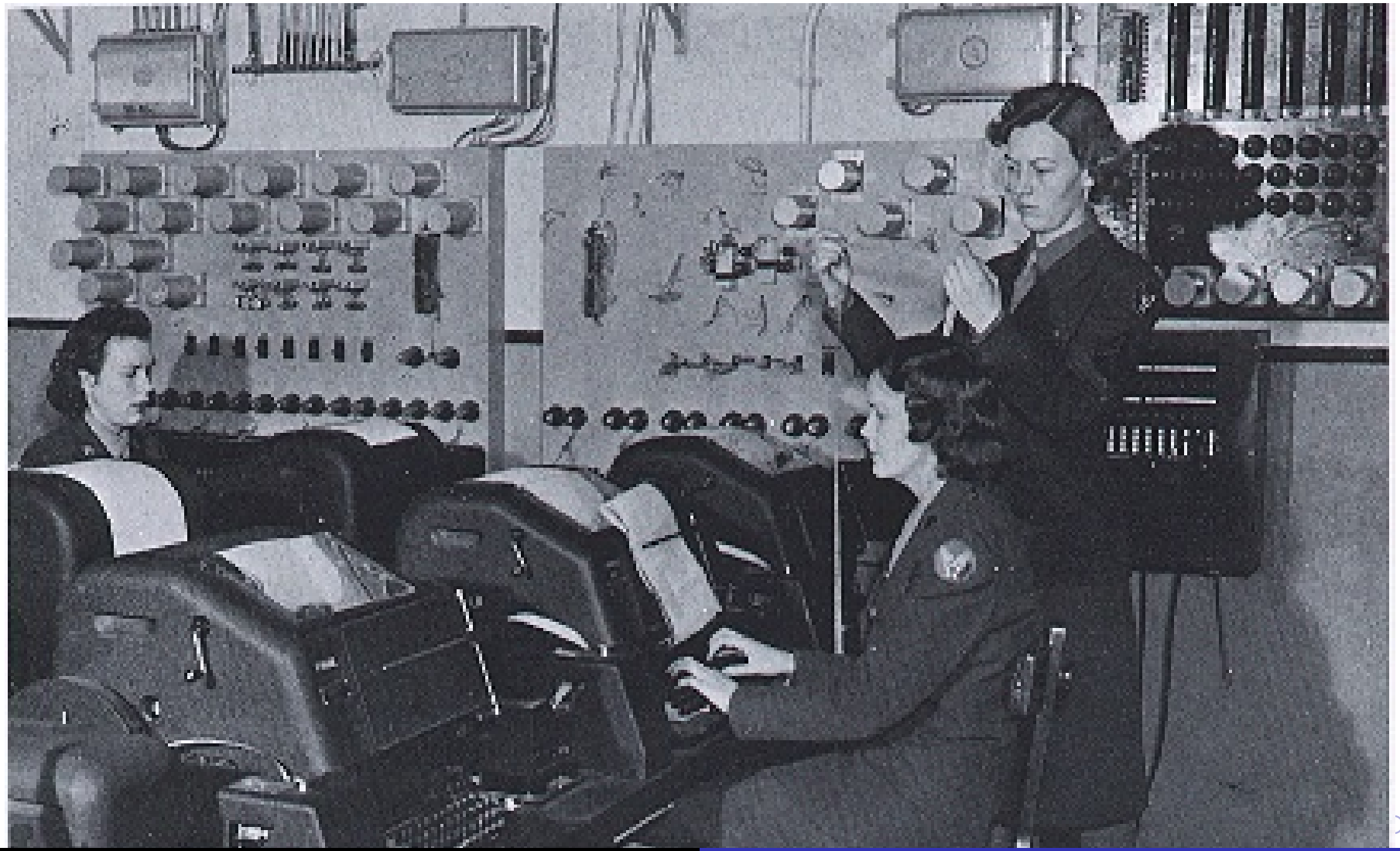
Cryptography is the study of "secret writing." This is the only branch of mathematics to be designated by the U.S. government as export-controlled. Cryptographic knowledge is considered to be "war materials!" While we won't head off into TOP SECRET territory we will have a bit of fun working out how to make (and to break) good secret codes.

the enigma



WACs



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practically speaking, they are synonyms...

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Alice (the sender) wants to send a message to. . .
Bob (the recipient) but they are afraid that. . .
Eve (the eavesdropper) will snoop on them and learn their secrets.

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decryption is the reverse process.
cryptanalysis is what Eve has to do in order to "break the code".

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Usually a **key** must also be chosen (in advance) by Alice and Bob.
If Eve knows the cryptosystem, she can attempt a "brute force" attack – try every possible key...

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cribs

If you have some idea what an encrypted message may be about, this allows you to make a list of "cribs." Cribs are words or phrases that may be part of the plaintext.

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It has been claimed that the Al Qaeda network hides messages in the low bits of pixels in internet porn.

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Dwwdfn dw gdzq rq iulgdb

Activity I

Write a creative message (not too long please. . .) and encrypt it with the Caesar shift.
Trade with another group and decrypt their message.

shift ciphers

The Caesar shift is the basis of the simple cryptosystem known as the shift cipher.
The key in a shift cipher is the amount of shifting that we will do to encode a message.
(For the original Caesar shift the key is $k = 3$.)

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It's not really that hard to just try all the possibilities and see if any of them look intelligible.

Activity II

Pick a key – this should be a relatively small integer. Lets keep things in the range -5 to 5.
Write a creative message and encrypt it with the shift cipher using your key.
Trade with another group and decrypt their message.

too easy

To get a useful cryptosystem we will need to develop a scheme where there are many more keys!

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Suppose it's zero o'clock. (You can continue to call it 12 if you want, but 0 is really more sensible.)
What time will it be after 5 five hour time periods go by?
So $5 \cdot 5 = 1$.

some practice

$$11 + 2 =$$

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$$11 + 2 = 1$$
$$7 \cdot 5 =$$

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$$11 + 2 = 1$$
$$7 \cdot 5 = 35$$

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$$7 \cdot 5 = 35 = 36 - 1$$

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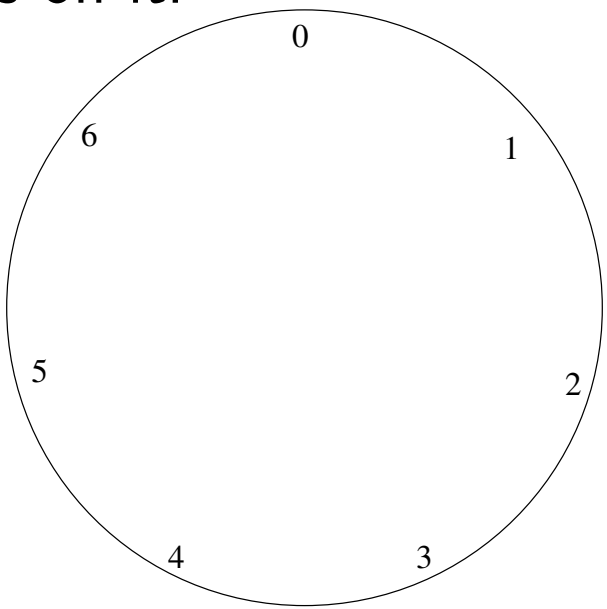
$$11 \cdot 11 = 121 = 1$$
$$6 \cdot 8 =$$

some practice

$$11 + 2 = 1$$
$$7 \cdot 5 = 35 = 36 - 1 = 11$$
$$11 \cdot 11 = 121 = 1$$
$$6 \cdot 8 = 48 = 0$$

clocks on other planets

A clock with 7 hours on it:



mod 7 operations

$4 + 5 =$

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$$4 + 5 = 2$$

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$6 + 4 =$

mod 7 operations

$$4 + 5 = 2$$
$$6 + 4 = 3$$

mod 7 operations

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$$3 \cdot 4 =$$

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Notice that the zero product property holds, since 7 is prime.

mod 26 operations and the alphabet

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The Caesar shift can now be described mathematically:

$$x \longrightarrow x + 3 \pmod{26}$$

The general shift cipher with key k is:

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Multiplying, on the other hand...
The trouble arises because $26 = 2 \cdot 13$ so if we avoid numbers that have either 2 or 13 as factors life will be good.

the affine cipher

In the affine cipher we encrypt using the map

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The keyspace contains $12 \cdot 26 = 312$ elements.
Suddenly a brute force approach is looking less enticing.

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The following message was encrypted with an affine cipher where the key was $m = 3$ and $b = 2$. I also took out spaces and punctuation (as is typical of encrypted ciphertext). What does it say?

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CBOQOXCNAPURKPWOH

duoncfzimepghvrxtjsbwlyqak

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$$26! = 26 \cdot 25 \cdot 24 \cdot \ldots \cdot 3 \cdot 2 \cdot 1 \approx 4.03 \times 10^{26}$$

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Kind of a lot of those keys would be weak, in the sense that too many letters would be encrypted as themselves. But even if we restrict to only those permutations where every letter gets moved there are plenty of keys.

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With a sufficiently large sample of ciphertext we can use an analysis of the frequency that the symbols occur to guess (accurately) about what the decryptions of certain symbols are.

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In this cipher we return to simply shifting the symbols of our plaintext up in the alphabet, but each symbol is shifted by a different amount.
Each time we run into an E it will get shifted to some other letter – but a different one each time! Frequency analysis will no longer work.

lemonlemonlemonlemonlemonlemonlem

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The key's letters tell you how much to shift.

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The original implementations of the Vigenère cipher involved using a word or a short phrase as the key.
The key's letters tell you how much to shift.
The key would be repeated as often as necessary so as to produce shift amounts for all of the letters in the plaintext.

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If you can figure out the length of the key you can break a Vigenere cipher into a bunch of parallel shift ciphers
Each of those is easy to break seperately.

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However, if we make the key for a Vigenere-type cipher be an arbitrarily long random sequence of letters we will have secure communication.
This is called a "one time pad"
There are certain places where you do not want to be caught with an arbitrarily long random sequence of letters about your person.

thanks!

Thank for coming, I hope you had fun!
http://www.nsa.gov/kids/